ASSIGMENT OF MAXIMAL ACCELERATION OF A CAR AS A FUNCTION OF SOME INDEPENDENT VARIABLES

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Abstract

The procedure for calculation of a car maximal acceleration during speeding up with changing shifts of a gearbox in dependence on several independent parameters has been presented in the paper. The parameters taken into consideration are the angle of a road slope and the additional vehicle load. The maximal acceleration for investigated vehicle category has been calculated on the basis of many numerical simulations. During these simulations the dynamics of vehicle motion has been modelled for different road conditions. In order to approximate the results of acceleration the power polynomials were applied. The least square method has been used in calculations of polynomial coefficients. The results of modelling of the vehicles dynamics concern two passenger cars and one heavy duty car. Simulations of speeding up were obtained using the model of vehicle dynamics.

Especially, model of vehicle, the traction characteristic of the passenger car with SI engine, the maximal driving force on wheels during vehicle acceleration calculated on the basis of traction characteristic, the maximal acceleration of the passenger cars and according to the model of vehicle dynamics during speeding up with changing shifts of a gearbox in dependence on different parameters of road slope and vehicle load are presented in the paper.

Keywords*: transport, modelling, vehicle dynamics, traffic flow*

1. Introduction

Appropriate modelling of traffic flow is one of the basic problems concerning identification and analysis of complex processes which take place in road networks. The modelling can be realized taking into account the rules influencing on every vehicle (microscopic models) or by treatment traffic as a continues stream of vehicles, which is characterized by traffic density and average speed (macroscopic models) [1, 2, 7].

The microscopic models are applied in case when dynamics of vehicles is a key factor, usually in study of traffic flow according to traffic signals. Parameters influencing vehicle dynamics (maximum acceleration and deceleration) have to be known in order to evaluate real traffic using the microscopic models. In contrary, the macroscopic traffic models are usually applied in computers systems in which the modelling of traffic flow is carried out for larger special and time scales. The time of calculations is key factor which determines the usefulness of the traffic macroscopic models.

In microscopic approach, appropriate parameterization of vehicle dynamics is still the main problem in modelling the traffic. The paper presents the description of method allowing defining maximum acceleration of the vehicle during speeding up whit changing shifts of a gearbox. The maximum acceleration is formulated as the function of some independent parameters. The proposed model allows generalizing the results of individual simulation and can be used in order to design maximal accelerations of vehicles. The presented algorithms, when applied, define the limits of vehicle accelerations in the microscopic traffic model as well.

2. The model of vehicle dynamics

The modelling of movement of multi body vehicles is an object of interests in many scientific centres connected with the motor industry. In spite of appearing commercial packages such as MSC.ADAMS and DADS, which are used to modelling the dynamics of complex systems and had specialized modules addressed to the simulation of kinematics and the dynamics of vehicles, the own non-commercial models are still formulated and applied. The main reason for that are different applications of the models and requirements according to the level of details. Moreover, specialized models, suitably formulated to solve a particular problem, can be easy involved to other computer packages, e.g. they can be elements of packages which allow modelling the influence of traffic on the environment [3].

The modelling of vehicle dynamics can be classified according to degree of details taken into consideration in proposed model. The most general motion model is a spatial model which allows to analyse the vehicle displacement in xyz direction. If the motion of vehicle is limited to the chosen plane e.g. xy, xz, yz then such kind of models can belong to the group of flat models. The motion models can be complemented with additional elements or sub-models, such as: the suspensions system, the wheels and the steering system. Driving torque and the road reaction forces on the wheels have important influence on vehicle dynamics. Different models of contact of tyre with the surface are in use. These models on the base of measurements define of contact forces in dependence on propriety of the tyres and the roadways. The Pacejka model is often used and applied in commercial packages [4]. The simpler model is the Dugoff-Uffelman model. The knowledge of contact forces between a road and tyre and coordinates of the points where they are applied allows calculating the speed and acceleration of vehicle.

The complex models (the spatial models) which have many degrees of freedom describe the movement of vehicle property and allow reflecting the real movement of vehicle. However, when these kind of models are used the time of numerical simulations are large. Simple models (the models of projection on plane) usually are being characterized by better numerical efficiency. Moreover, application of models with a large degree of sophistication is not always well-founded. Such regularities are observed in the case of traffic flow modelling in road networks, e. g. for the qualification of emission of car exhaust pollutants.

In order to calculate the maximal acceleration of a vehicle the model of vehicle dynamics in rectilinear motion (the motion xz plane) is formulated and sloved. In the paper [4] the model of the vehicle dynamics is described in details. Its main component elements are (Fig. 1):

- the car body for which is possible to analyse the displacement in horizontal and vertical direction and angle of rotation (three degrees of freedom),
- the suspensions, having one degree of freedom in vertical direction in motion in relation to the car body,
- the wheels, with one degree of freedom in relative motion.

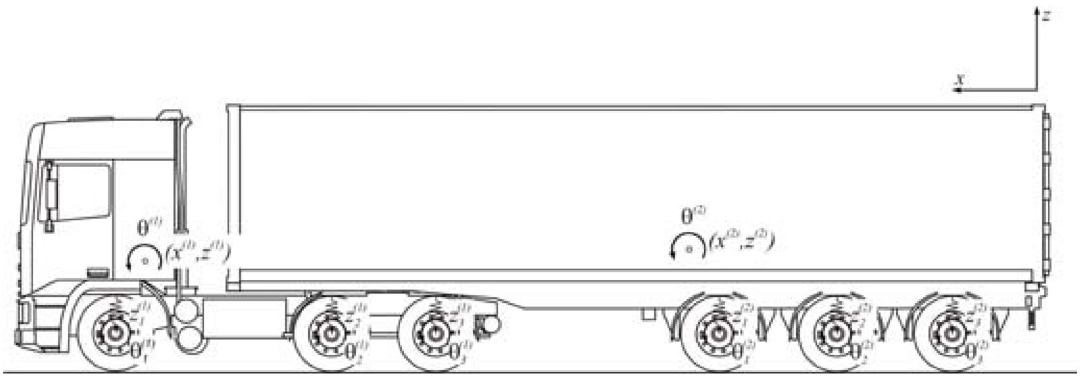


Fig. 1. Model of vehicle

The motion of vehicle is described using seven generalized coordinates in case of passenger car and eighteen for truck with semitrailer (Fig. 1.). The equations of motion are derived from the Lagrange equations. The vector of generalized forces takes into account following: the gravity force, the forces acting on wheels from road surface and the air resisting force. The equations of single element (i) can expressed as follows:

$$\mathbf{M}^{(i)} \cdot \ddot{\mathbf{q}}^{(i)} = \mathbf{F}^{(i)}\left(t, \mathbf{q}^{(i)}, \dot{\mathbf{q}}^{(i)}, \mathbf{F}_Q^{(i)}, \mathbf{F}_R^{(i)}, \mathbf{F}_W^{(i)}, \mathbf{M}_n^{(i)}\right), \tag{1}$$

where:

$\mathbf{q}^{(i)}$ - is the vector of generalized coordinates of a car body, suspensions and wheels,

$\mathbf{M}^{(i)}$ - is the mass matrix,

$\mathbf{F}^{(i)}$ - is the vector of generalized forces ($\mathbf{F}_Q^{(i)}$ – the vector of gravity forces, $\mathbf{F}_R^{(i)}$ – the vector of road surface reaction forces, $\mathbf{F}_W^{(i)}$ – the vector of air resistance forces),

$\mathbf{M}_n^{(i)}$ - is the vector of wheel driving torques.

To integrate individual elements in multi-body vehicles, all elements have been connected by joints in the kinematic chain. This is realized by formulation suitable constraint equations involving reaction forces to equations of motion for individual elements. Taking into consideration constraint equations, the equations which describe the motion of whole system can be presented on the form:

$$\begin{cases} \mathbf{M} \cdot \ddot{\mathbf{q}} + \mathbf{DR} = \mathbf{F}, \\ \mathbf{D}^T \cdot \ddot{\mathbf{q}} = \mathbf{W}, \end{cases} \tag{2}$$

where:

D - is the coefficient matrix,

R - is the vector of reaction forces.

In solution of arrangement of ordinary differential equations usually one from following methods of integration can be applied:

- in case of the constant step size of integration: the higher order Runge-Kutta methods and the extrapolation-interpolation methods,
- in case of the variable step size of integration: the Runge-Kutta-Fehlberg metod or the Rosenbrock method or the Bulirscha-Stoer method and the Bulirsch-Stoer-Daufhardt metod.

Their detailed description and the comparison of numeric efficiency in the questions of modelling of vehicle dynamics can be found in papers [5, 6, 8].

Calculation of the maximal vehicle acceleration during speeding up with changing shifts of a gearbox is connected with calculation of a maximum driving moment which can be reach on driving wheels. This moment can be directly determined from the traction characteristic, where driving forces depending on the vehicle speed. Calculation of driving force that provides the highest acceleration, can be achieve from graph $F_n = f(v)$. The example of traction profile for passenger car with SI engine is shown in Fig. 2a. The maximum driving force calculated for this traction profile is presented in Fig. 2b.

When a vehicle maximum acceleration is designed, the attention should be devoted to the situation, in which torque on driven wheels is too high. In that case the wheel can come to break of the adherence (the wheel spin). Such situation is unfavourable and does not allow getting higher acceleration by vehicle, reduces its steer ability and increases of the consumption of tyres directly. To prevent these unfavourable phenomena (the slide) the border condition of longitudinal slide forces should be applied. The detailed description of the mathematical model of the vehicle dynamics, limitation of the maximal moments on wheels which allow eliminating their sliding and results of the model validation is presented in paper [3].

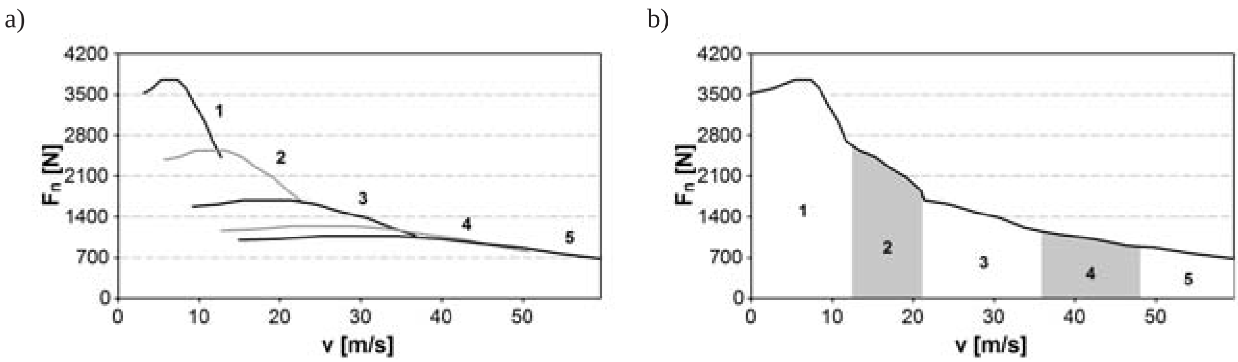


Fig. 2. a) The traction characteristic of the passenger car with SI engine, b) the maximal driving force on wheels during vehicle acceleration calculated on the basis of traction characteristic

3. The modelling of vehicle motion

The identification of vehicle dynamic properties (acceleration, deceleration) can be realised for the various set of independent parameters using the vehicle dynamic model. Among parameters which directly affect on vehicle acceleration, the main role play the road conditions (the friction coefficients describing roadway and the angle of road slope) and the vehicle mass (the additional load). In order to illustrate the influences of this quantities on the maximal acceleration during the manoeuvre of speeding up some numerical simulations of the motion of two vehicles were carried out which belongs to the passenger car category (PC).The first had SI engine (SI), the second one CI engine (CI). Both passenger cars had a fifth shifts gearbox.

In Fig. 3-5 are shown influences of the type of a road surfaces, the angle of road slope and the additional vehicle load on accelerations of vehicles PC-SI and PC-CI calculated using the presented model of vehicle dynamics.

Analysis of the results shows that maximal vehicle acceleration strongly depends on road surface slope and slightly depend on road surface and additional load. In further considerations the dependence of maximal acceleration of passenger car on road surfaces could be omitted.

4. The parameterization of maximal acceleration in microscopic flow models

The basic parameter taken into account in the microscopic flow models having fundamental influence on vehicle dynamics is the maximal acceleration. As it was shown in previous section in the real traction exploitation conditions the value of the maximal acceleration depends on some independent parameters.

In order to parameterize the microscopic models, one has to known functions describing the maximal acceleration for given values of independent parameters for any vehicle categories. For each vehicle category, the function describing the maximal acceleration can be obtained by generalization of the results of a computer simulation using one of approximation methods. In the paper we assume that function approximating the maximal vehicle acceleration according to parameters $x_1, \ldots, x_r$ has the following general form:

$$a = \sum_{i_1=0}^{n_1} \ldots \sum_{i_j=0}^{n_j} \ldots \sum_{i_r=0}^{n_r} \alpha_{i_1,\ldots,i_j,\ldots,i_r} x_1^{i_1} \ldots x_j^{i_j} \ldots x_r^{i_r}, \tag{3}$$

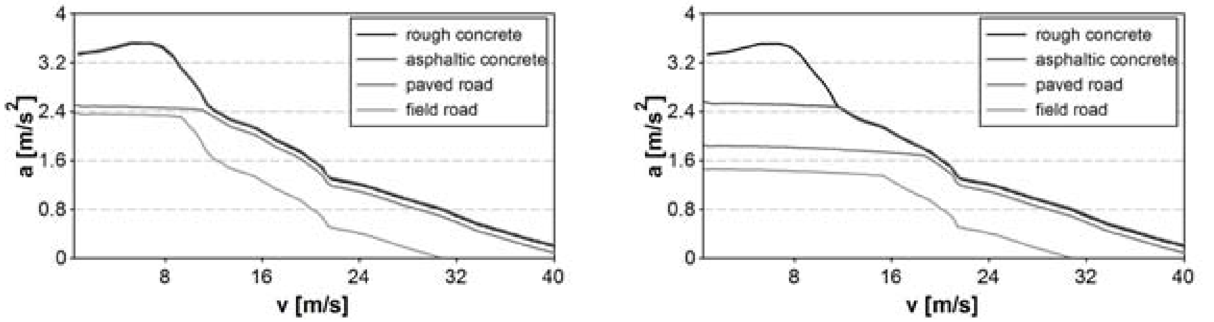
where:

$n_1, \ldots, n_r$ - are the degrees of polynomial,

$\alpha_1, \ldots, \alpha_r$ - are the polynomial coefficients.

dry road surface wet road surface

a) PC-SI



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b) PC-CI

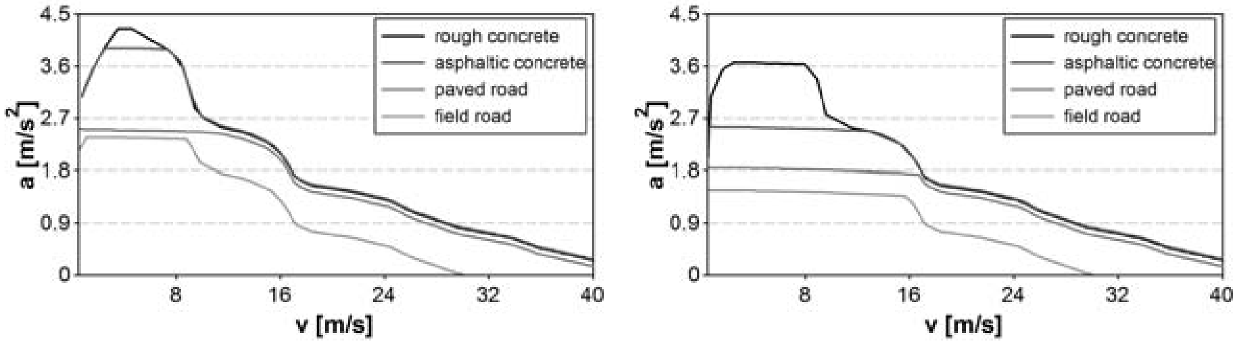

Fig. 3. The maximal acceleration a [m/s²] of the passenger cars a) PC-SI and b) PC-CI according to the model of vehicle dynamics during speeding up with changing shifts of a gearbox in dependence on different type of road surface

a) PC-SI b) PC-CI

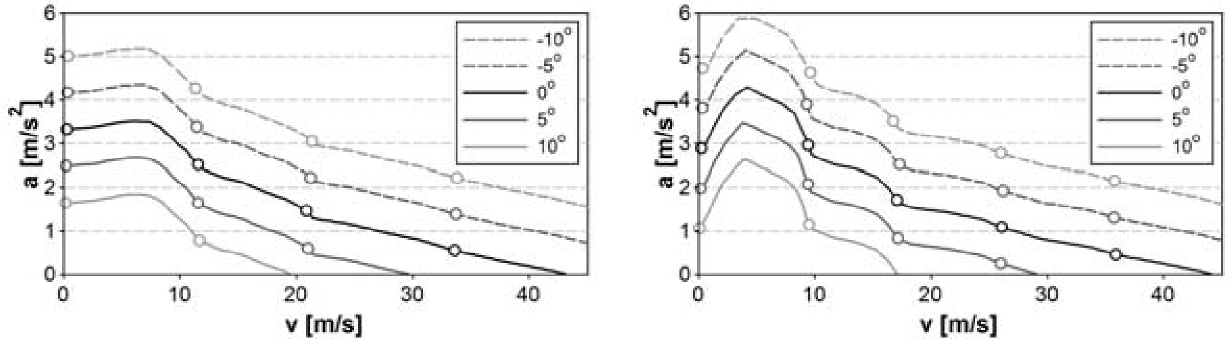

Fig. 4. The maximal acceleration a [m/s²] of the passenger cars a) PC-SI and b) PC-CI according to the model of vehicle dynamics during speeding up with changing shifts of a gearbox in dependence on the angle of road slope

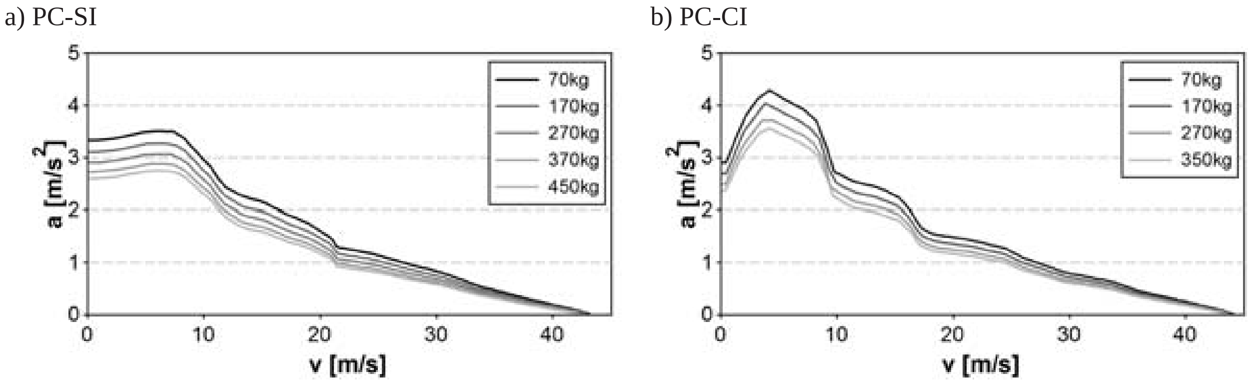

Fig. 5. The maximal acceleration a [m/s²] of a passenger cars a) PC-SI and b) PC-CI according to the model of vehicle dynamics during speeding up with changing shifts of a gearbox in dependence on the additional vehicle load

The coefficient α_1,..., α_r have been calculated using the least square methods [6]. The example of calculation results is presented bellow. The numerical simulation of speeding up manoeuvre was performed for three different vehicles:

- the passenger car with a SI engine (V1),
- the passenger car with a CI engine (V2),
- the heavy duty car with a semitrailer (V3).

Three parameters influencing acceleration has been taken into consideration: the vehicle speed (x_1),the angle of road scope (x_2) and the additional vehicle load (x_3).

Next, the approximation polynomials coefficients have been calculated, assuming:

- for the passenger cars (V1, V2): r = 3, n1 $\in\{4,6,8\}$, n2 = n3 $\in\{1,2\}$,
- for the heavy duty car (V3): r = 1, n1 $\in\{4,6,8\}$, n2 $\in\{1,2\}$.

The approximation error E is defined by formulae:

$$E = \frac{1}{N}\sum_{k=1}^{N}\left|\frac{a\left(x_1^{(k)},...,x_r^{(k)}\right)-a_k}{a_k}\right|100\%, \tag{4}$$

where N is the number of discrete values of acceleration obtained from the model.

The approximation errors according to different values of polynomial degrees for considered vehicles are presented in Tab. 1.

Tab. 1. The approximation error E and the number n_α of power polynomial degrees

n_1	n_2, n_3	V1; N=3238	V2; N=2948	V3; N=637
4	1	7.55	5.23	4.81
4	2	7.35	5.10	4.77
6	1	3.38	4.46	2.60
6	2	3.21	4.38	2.58
8	1	2.79	3.88	1.14
8	2	2.60	3.71	1.11

The comparison of approximation errors, allows to state that the correct description of the acceleration process is achieved when for variable x_1 (the car speed) the degree of polynomial n_1 is larger than 4. Increasing n_2 and n_3 above value 1 does not provide to significant improvement of the approximation errors. The maximum acceleration for considered vehicles categories is approximated with an acceptance error for polynomials with n_1 = 6 and n_2 = n_3 = 1.

The surfaces (calculated from 3) describing the maximal acceleration of passenger vehicles V1 and V2 in dependence on the vehicle speed v and the angle of road slope α are presented in Fig. 6. Fig. 7 shows how maximal acceleration depends on the vehicle speed v and the additional vehicle load m_d, while in Fig. 8 one can observe how the maximal acceleration of heavy duty vehicle V3 depends on v and α.

a) b)

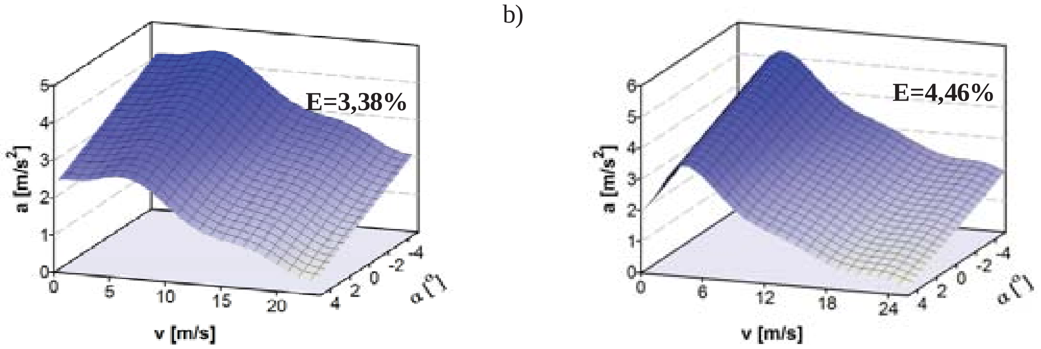

Fig. 6. The maximal acceleration a[m/s^2] of the passenger cars a) V1 and b) V2 during speeding up with changing shifts of a gearbox according to the vehicle velocity v [m/s] and the angle of road slope α [^{0}]

a) b)

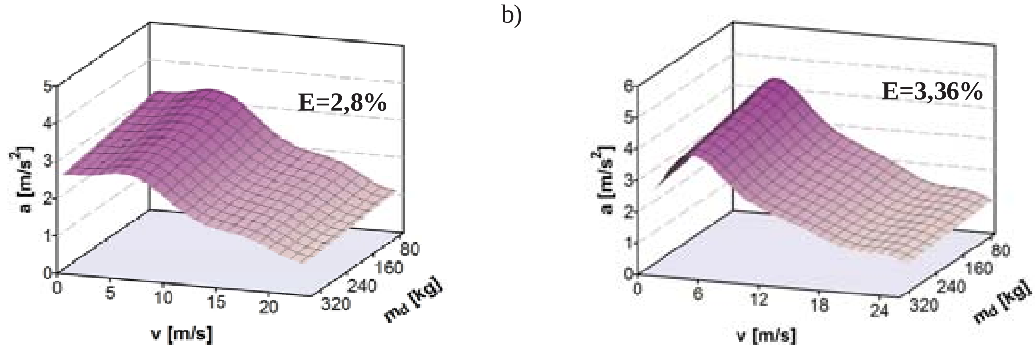

Fig. 7. The maximal acceleration a [m/s^2] of the passenger cars a) V1 and b) V2 during speeding up with changing shifts of a gearbox according to the vehicle velocity v [m/s] and an additional vehicle load m_d [kg]

a) b)

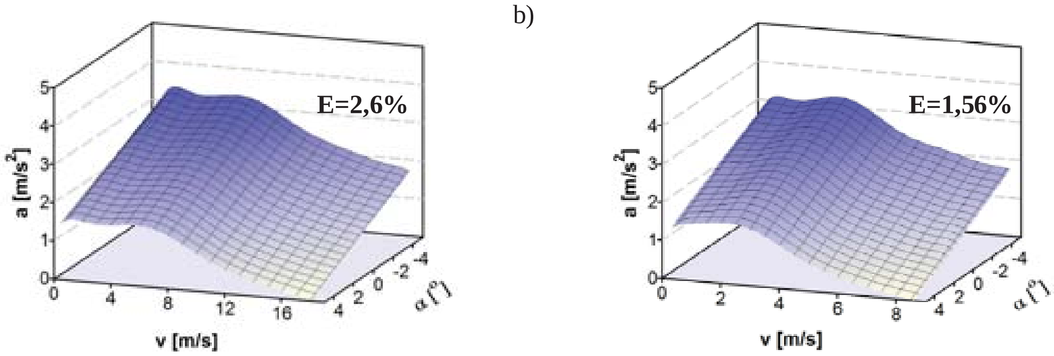

Fig. 8. The maximal acceleration a [m/s^2] of the heavy duty car a) with not loaded semitrailer and b) with loaded semitrailer during speeding up according to the vehicle velocity v [m/s] and the angle of road slope α [^{0}]

These approximation surfaces can be directly used in microscopic flow models as a limitation for vehicle motion. The approximation errors are below 5% in each case.

5. Conclusions

The modelling of traffic flow in the road networks taken into consideration the dynamics of moving vehicles, requires the suitably parameterized microscopic flow models. The model of multi-body vehicle dynamics presented in the paper allows calculating the maximal vehicle acceleration during speeding up with changing shifts of a gearbox in dependence on different road and meteorological conditions. The realization of many simulations with the model of vehicle motion allows to create a data base. The data collected in that base concerns detailed parameters of vehicle dynamics. In the paper the power polynomials functions were proposed in order to generalize results of simulations. Analysis of approximation errors shows that method of generelization allows modelling with acceptance accuracy. Moreover, the method could be applied as the limitation in microscopic traffic flow models. The advantage of proposed procedure is the possibility of identification of maximal acceleration for vehicles belonging to various categories and possessing a different specification of engine and gearbox. The applied model of vehicle dynamics after the validation makes the alternative for expensive road investigations [3].

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